Többváltozós lineáris regresszió

Mátrixműveletek

$$k*A = \begin{vmatrix} k*a_{11} & k*a_{12} \\ k*a_{21} & k*a_{22} \end{vmatrix}$$

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$B = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$$

$$A + B = \begin{vmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{vmatrix}$$

$$A^T = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}$$

$A = B$?

$$a_{11} = b_{11} \quad a_{12} = b_{12}$$

$$a_{21} = b_{21} \quad a_{22} = b_{22}$$

Mátrixműveletek

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad B = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$$

$$A * B = \begin{vmatrix} a_{11}*b_{11}+a_{12}*b_{21} & a_{11}*b_{12}+a_{12}*b_{22} \\ a_{21}*b_{11}+a_{22}*b_{21} & a_{21}*b_{12}+a_{22}*b_{22} \end{vmatrix}$$

Mátrixműveletek

Osztás? **Inverz mátrixxal szorzás**

$A * A^{-1} = I$ **Egységmátrix** $I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad A^{-1} = \begin{vmatrix} \dfrac{a_{22}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{-\ a_{12}}{a_{11}*a_{22}-a_{12}*a_{21}} \\ \dfrac{-\ a_{21}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{a_{11}}{a_{11}*a_{22}-a_{12}*a_{21}} \end{vmatrix}$$

$$A * A^{-1} = \begin{vmatrix} \dfrac{a_{11}*a_{22}-a_{12}*a_{21}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{a_{11}*a_{12}-a_{11}*a_{12}}{a_{11}*a_{22}-a_{12}*a_{21}} \\ \dfrac{a_{21}*a_{22}-a_{21}*a_{22}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{a_{11}*a_{22}-a_{12}*a_{21}}{a_{11}*a_{22}-a_{12}*a_{21}} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

Lineáris regresszió - mátrix

$$\boxed{\begin{aligned} y_1 &= a + b*x_1 \\ y_2 &= a + b*x_2 \\ y_3 &= a + b*x_3 \end{aligned}}$$

$$Y = X*B$$

$$Y = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix} \qquad X = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{vmatrix} \qquad B = \begin{vmatrix} a \\ b \end{vmatrix}$$

$$Y = X*B = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{vmatrix} * \begin{vmatrix} a \\ b \end{vmatrix} = \boxed{\begin{vmatrix} a + b*x_1 \\ a + b*x_2 \\ a + b*x_3 \end{vmatrix} = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix}}$$

Lineáris regresszió - mátrix

$$\boxed{HSQ = \sum_{i=1}^{n}\left(y_i - y_{szi}\right)^2} \quad \textbf{-> minimum (a,b)}$$

$$\mathbf{HSQ = (Y\text{-}Ysz)^2 = (Y\text{-}Ysz)^T * (Y\text{-}Ysz)}$$

$$(Y\text{-}Ysz)^T = \begin{vmatrix} y_1\text{-}y_{sz1} & y_2\text{-}y_{sz2} & y_3\text{-}y_{sz3} \end{vmatrix} \qquad Y\text{-}Ysz = \begin{vmatrix} y_1\text{-}y_{sz1} \\ y_2\text{-}y_{sz2} \\ y_3\text{-}y_{sz3} \end{vmatrix}$$

$$(Y\text{-}Ysz)^T * (Y\text{-}Ysz) = \begin{vmatrix} y_1\text{-}y_{sz1} & y_2\text{-}y_{sz2} & y_3\text{-}y_{sz3} \end{vmatrix} * \begin{vmatrix} y_1\text{-}y_{sz1} \\ y_2\text{-}y_{sz2} \\ y_3\text{-}y_{sz3} \end{vmatrix} =$$

$$= (y_1\text{-}y_{sz1})^2 + (y_2\text{-}y_{sz2})^2 + (y_3\text{-}y_{sz3})^2 = \boxed{\sum_{i=1}^{n}\left(y_i - y_{szi}\right)^2}$$

Lineáris regresszió - mátrix

$Ysz = X*B$ $\boxed{Y^T*Ysz = Ysz^T*Y}$

$$HSQ = (Y-Ysz)^2 = (Y-Ysz)^T*(Y-Ysz) =$$

$$= Y^T*Y - \boxed{Y^T*Ysz} - \boxed{Ysz^T*Y} + Ysz^T*Ysz =$$

$$= Y^T*Y - 2*\boxed{Ysz^T*Y} + Ysz^T*Ysz =$$

$$HSQ = Y^T*Y - 2*B^T*X^T*Y + B^T*X^T*X*B$$

$$\frac{dHSQ}{dB} = -2 \cdot X^T \cdot Y + 2 \cdot X^T \cdot X \cdot B = 0$$

$$-X^T*Y + X^T*X*B = 0$$

$$X^T*X*B = X^T*Y$$

$$B = (X^T*X)^{-1}*(X^T*Y)$$

Keresztszorzat mátrix SP

$$A = \begin{vmatrix} x1 - \text{xátl} & y1 - \text{yátl} \\ x2 - \text{xátl} & y2 - \text{yátl} \\ x3 - \text{xátl} & y3 - \text{yátl} \\ x4 - \text{xátl} & y4 - \text{yátl} \\ x5 - \text{xátl} & y5 - \text{yátl} \end{vmatrix}$$

$$A^T * A$$

$$\begin{vmatrix} x1\text{-xátl} & x2\text{-xátl} & x3\text{-xátl} & x4\text{-xátl} & x5\text{-xátl} \\ y1\text{-yátl} & y2\text{-yátl} & y3\text{-yátl} & y4\text{-yátl} & y5\text{-yátl} \end{vmatrix} * \begin{vmatrix} x1\text{-xátl} & y1\text{-yátl} \\ x2\text{-xátl} & y2\text{-yátl} \\ x3\text{-xátl} & y3\text{-yátl} \\ x4\text{-xátl} & y4\text{-yátl} \\ x5\text{-xátl} & y5\text{-yátl} \end{vmatrix}$$

Keresztszorzat mátrix SP $A^T * A$

$$\begin{vmatrix} x1\text{-}x\acute{a}tl & x2\text{-}x\acute{a}tl & x3\text{-}x\acute{a}tl & x4\text{-}x\acute{a}tl & x5\text{-}x\acute{a}tl \\ y1\text{-}y\acute{a}tl & y2\text{-}y\acute{a}tl & y3\text{-}y\acute{a}tl & y4\text{-}y\acute{a}tl & y5\text{-}y\acute{a}tl \end{vmatrix} * \begin{vmatrix} x1\text{-}x\acute{a}tl & y1\text{-}y\acute{a}tl \\ x2\text{-}x\acute{a}tl & y2\text{-}y\acute{a}tl \\ x3\text{-}x\acute{a}tl & y3\text{-}y\acute{a}tl \\ x4\text{-}x\acute{a}tl & y4\text{-}y\acute{a}tl \\ x5\text{-}x\acute{a}tl & y5\text{-}y\acute{a}tl \end{vmatrix}$$

$$SQ_x = \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2$$

$$SP_{xy} = \sum_{i=1}^{n} \left(x_i - \overline{x}\right) \cdot \left(y_i - \overline{y}\right)$$

$$SP = \begin{vmatrix} SQx & SPxy \\ SPxy & SQy \end{vmatrix}$$

Korrelációs mátrix R

$$\mathbf{AR} = \begin{vmatrix} (x1-xátl)/gyök(SQx) & (y1-yátl)/gyök(SQy) \\ (x2-xátl)/gyök(SQx) & (y2-yátl)/gyök(SQy) \\ (x3-xátl)/gyök(SQx) & (y3-yátl)/gyök(SQy) \\ (x4-xátl)/gyök(SQx) & (y4-yátl)/gyök(SQy) \\ (x5-xátl)/gyök(SQx) & (y5-yátl)/gyök(SQy) \end{vmatrix}$$

$$\text{ÖSQ} = \text{SQy} = \sum_{i=1}^{n} \left(y_i - \overline{y}\right)^2$$

$$\text{SP}_{xy} = \sum_{i=1}^{n} \left(x_i - \overline{x}\right) \cdot \left(y_i - \overline{y}\right)$$

$$\mathbf{R = AR^T * AR =} \begin{vmatrix} \mathbf{1} & \mathbf{r_{xy}} \\ \mathbf{r_{xy}} & \mathbf{1} \end{vmatrix}$$

$$\text{RSQ} = \text{SP}^2/\text{SQx}$$

$$\mathbf{R^2 = RSQ/ÖSQ =} \frac{\text{SP}^2}{\text{SQx} \cdot \text{SQy}}$$

$$r_{xy} = \frac{SP}{\sqrt{SQx} \cdot \sqrt{SQy}}$$

Variancia táblázat						det.koeff
		SQ	**FG**	**MQ**	**Farány**	r^2
Összes	SQy	3,5	2	1,75	RMQ/HMQ	RSQ/ÖSQ
Regresszió	SP^2/SQx	2	1	2	1,3	0,5714
Hiba	ÖSQ-RSQ	1,5	1	1,5	39,9	
					Korrelációs koeff: r =	0,7559