Többváltozós lineáris regresszió

Mátrixműveletek

$$k^*A = \begin{vmatrix} k^*a_{11} & k^*a_{12} \\ k^*a_{21} & k^*a_{22} \end{vmatrix}$$

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$B = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$$

$$A + B = \begin{vmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{vmatrix}$$

$$A^T = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}$$

$$A = B \ ? \quad \begin{matrix} a_{11} = b_{11} & a_{12} = b_{12} \\ a_{21} = b_{21} & a_{22} = b_{22} \end{matrix}$$

Mátrixműveletek

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad B = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$$

$$A * B = \begin{vmatrix} a_{11}*b_{11}+a_{12}*b_{21} & a_{11}*b_{12}+a_{12}*b_{22} \\ a_{21}*b_{11}+a_{22}*b_{21} & a_{21}*b_{12}+a_{22}*b_{22} \end{vmatrix}$$

Mátrixműveletek

Osztás? **Inverz mátrixxal szorzás**

$A * A^{-1} = I$ **Egységmátrix** $I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad A^{-1} = \begin{vmatrix} \dfrac{a_{22}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{-\,a_{12}}{a_{11}*a_{22}-a_{12}*a_{21}} \\ \dfrac{-\,a_{21}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{a_{11}}{a_{11}*a_{22}-a_{12}*a_{21}} \end{vmatrix}$$

$$A * A^{-1} = \begin{vmatrix} \dfrac{a_{11}*a_{22}-a_{12}*a_{21}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{a_{11}*a_{12}-a_{11}*a_{12}}{a_{11}*a_{22}-a_{12}*a_{21}} \\ \dfrac{a_{21}*a_{22}-a_{21}*a_{22}}{a_{11}*a_{22}-a_{12}*a_{21}} & \dfrac{a_{11}*a_{22}-a_{12}*a_{21}}{a_{11}*a_{22}-a_{12}*a_{21}} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

Lineáris regresszió - mátrix

$$\boxed{\begin{aligned} y_1 &= a + b*x_1 \\ y_2 &= a + b*x_2 \\ y_3 &= a + b*x_3 \end{aligned}}$$

$$Y = X*B$$

$$Y = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix} \qquad X = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{vmatrix} \qquad B = \begin{vmatrix} a \\ b \end{vmatrix}$$

$$Y = X*B = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{vmatrix} * \begin{vmatrix} a \\ b \end{vmatrix} = \boxed{\begin{vmatrix} a + b*x_1 \\ a + b*x_2 \\ a + b*x_3 \end{vmatrix} = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix}}$$

Lineáris regresszió - mátrix

$$\boxed{HSQ = \sum_{i=1}^{n} \left(y_i - y_{szi}\right)^2}$$ **-> minimum (a,b)**

$$\mathbf{HSQ = (Y\text{-}Ysz)^2 = (Y\text{-}Ysz)^T * (Y\text{-}Ysz)}$$

$$\mathbf{Y\text{-}Ysz} = \begin{vmatrix} y_1 - y_{sz1} \\ y_2 - y_{sz2} \\ y_3 - y_{sz3} \end{vmatrix} \qquad \mathbf{(Y\text{-}Ysz)^T} = \begin{vmatrix} y_1 - y_{sz1} & y_2 - y_{sz2} & y_3 - y_{sz3} \end{vmatrix}$$

$$\mathbf{(Y\text{-}Ysz)^T * (Y\text{-}Ysz)} = \begin{vmatrix} y_1 - y_{sz1} & y_2 - y_{sz2} & y_3 - y_{sz3} \end{vmatrix} * \begin{vmatrix} y_1 - y_{sz1} \\ y_2 - y_{sz2} \\ y_3 - y_{sz3} \end{vmatrix} =$$

$$= (y_1 - y_{sz1})^2 + (y_2 - y_{sz2})^2 + (y_3 - y_{sz3})^2 = \boxed{\sum_{i=1}^{n} \left(y_i - y_{szi}\right)^2}$$

Lineáris regresszió - mátrix

$$Ysz = X*B \qquad \boxed{Y^T*Ysz = Ysz^T*Y}$$

$$HSQ = (Y-Ysz)^2 = (Y-Ysz)^T*(Y-Ysz) =$$

$$= Y^T*Y - \boxed{Y^T*Ysz} - \boxed{Ysz^T*Y} + Ysz^T*Ysz =$$

$$= Y^T*Y - 2*\boxed{Ysz^T*Y} + Ysz^T*Ysz =$$

$$HSQ = Y^T*Y - 2*B^T*X^T*Y + B^T*X^T*X*B$$

$$\frac{dHSQ}{dB} = -2 \cdot X^T \cdot Y + 2 \cdot X^T \cdot X \cdot B = 0$$

$$- X^T*Y + X^T*X*B = 0$$

$$X^T*X*B = X^T*Y$$

$$B = (X^T*X)^{-1}*(X^T*Y)$$